COMPLEX ANALYSIS

Multiple Choice Questions

MODULE I

1. The principal argument of the complex number $-1-i$ is

   (a) $\frac{3\pi}{4}$

   (b) $-\frac{\pi}{4}$

   (c) $-\frac{3\pi}{4}$

   (d) $\frac{\pi}{4}$

2. Real part of $f(z)=\frac{1}{1-z}$ is

   (a) $\frac{1-x}{(1-x)^2+y^2}$

   (b) $\frac{1+x}{(1-x)^2+y^2}$

   (c) $\frac{1-x}{x^2+(1-y)^2}$

   (d) $\frac{1-x}{(1-x)^2-y^2}$

3. Polar form of the Cauchy- Riemann equation is

   (a) $\frac{\partial u}{\partial r}=r\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r}=-r\frac{\partial u}{\partial \theta}$

   (b) $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$

   (c) $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r}=-r\frac{\partial u}{\partial \theta}$

   (d) $\frac{\partial u}{\partial r}=r\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$

4. Which of the following is true?

   (a) Differentiability does not implies continuity

   (b) Differentiability implies continuity

   (c) Continuity implies differentiability

   (d) There is no relation between continuity and differentiability

5. The function $f(z)=|z|^2$ has

   (a) One singular point

   (b) Two singular points

   (c) Three singular points

   (d) No singular point

6. In the neighbourhood of $z = 1$, the function $f(z)$ has a power series expansion of the form $f(z) = 1 + (1 - z) + (1 - z)^2 + \cdots \infty$

   Then $f(z)$ is

   (a) $\frac{1}{z}$

   (b) $\frac{-1}{z^{-2}}$

   (c) $\frac{z-1}{z+1}$

   (d) $\frac{1}{2z-1}$

7. What is the value of m for which $2x - x^2 + my^2$ is harmonic ?

   (a) 1

   (b) -1

   (c) 2

   (d) -2

8. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane?

   (a) $f(z) = z^2$

   (b) $f(z) = e^z$

   (c) $f(z) = \sin z$

   (d) $f(z) = \log z$

9. Value of $(1 + i)^{24}$ is

   (a) 2^{24}

   (b) 2^{12}

   (c) 2^8

   (d) 2^2

10. If $f(z)$ is an analytic function whose real part is constant then $f(z)$ is

    (a) function of z

    (b) function of x only

    (c) function of y only

    (d) constant

11. A function which is analytic everywhere in a complex plane is known as

    (a) Harmonic function

    (b) differentiable function

    (c) regular function

    (d) entire function

12. The value of $\left|e^{i\theta}\right|$ is

    (a) 1

(b) 0

(c) -1

(d) π

13. The function $f(z) = xy + iy$ is

(a) Nowhere analytic

(b) Analytic every where

(c) Analytic only at origin

(d) Analytic except at the origin

14. The harmonic conjugate of $u(x, y) = \frac{y}{x^2+y^2}$ is

(a) $2xy + y + C$

(b) $xy + y + C$

(c) $2xy + 2y + C$

(d) $2xy - y + C$

15. Period of e^z is

(a) 2π

(b) -2π

(c) $2\pi i$

(d) π

16. If z is a non zero complex number, then for $n = 1, 2, 3, \ldots,\quad z^{1/n}$ is

(a) $exp(nlog\ z)$

(b) $exp\left(\frac{1}{n} log\ z\right)$

(c) $exp\left(\frac{1}{n} log\ \frac{1}{z}\right)$

(d) $exp\left(nlog\ \frac{1}{z}\right)$

17. The principal value of $(-i)^i$ is

(a) $\exp\frac{\pi}{4}$

(b) $\exp -\frac{\pi}{4}$

(c) $\exp -\frac{\pi}{2}$

(d) $\exp\frac{\pi}{2}$

18. $2\sin(z_1 + z_2)\sin(z_1 - z_2) =$

(a) $\cos 2z_2 + \cos 2z_1$

(b) $\cos 2z_2 - \cos 2z_1$

(c) $\cos 2z_1 + \cos 2z_2$

(d) $\cos 2z_1 - \cos 2z_2$

19. $|\sin az|^2 =$

(a) $sin^2\ ax + sinh^2 ay$

(b) $sin^2\ ax - sinh^2 ay$

(c) $cos^2\ ax + sinh^2 ay$

(d) $cos^2\ ax - sinh^2 ay$

20. Real part of the function $|z|^2$ is

(a) $x^2 - y^2$

(b) $2xy$

(c) $x^2 + y^2$

(d) $y^2 - x^2$

MODULE II

1. The value of the integral $\int_C \frac{dz}{z^2}$ where C is the positively oriented circle $z = 2e^{i\theta}$ $(-\pi < \theta \leq \pi)$ about the origin is

(a) 1

(b) 0

(c) -1

(d) 2

2. The integral of $\oint_C \frac{dz}{z^2+9}$, where C is the unit circle is

(a) 0

(b) 1

(c) 3

(d) -3

3. A domain that is not simply connected is said to be

(a) Contour

(b) multiply connected

(c) connected

(d) None of these

4. The value of the integral $\int_C \frac{z^2}{z-2}\ dz$, where C is the circle $|z| = 3$ is

(a) $2\pi i$

(b) $-\pi i$

(c) $4\pi i$

(d) $8\pi i$

5. If $P(z)$ is a polynomial of degree n $(n \geq 1)$ then it has

(a) $n + 1$ zeros

(b) n zeros

(c) $n-1$ zeros

(d) no zeros

6. The integral of the function $\int_0^1 (1+it)^2\, dt$ is

(a) $2+i$

(b) $\frac{1}{3}+i$

(c) $\frac{2}{3}+i$

(d) $1+i$

7. If a function f is analytic throughout a simple connected domain D, then $\int_C f(z)\, dz =$

(a) 0

(b) $2\pi i$

(c) $2\pi i\, f(z)$

(d) 1

8. The integral of the function $\int_C \frac{\cos z}{z}\, dz$ where C is the unit circle is

(a) π

(b) $-\pi i$

(c) πi

(d) $2\pi i$

9. The integral of the function $\int_C \frac{\cos z}{2z}\, dz$ where C is the unit circle is

(a) π

(b) $-\pi i$

(c) πi

(d) $2\pi i$

10. The integral of $\int_C \frac{dz}{z-i}$ where C is the circle $|z|=2$ is

(a) $2\pi i$

(b) $-\pi i$

(c) πi

(d) $-2\pi i$

11. The integral of the function $\int_C e^z \cos z\, dz$ where C is the unit circle is

(a) $\pi(3+2i)$

(b) $\frac{\pi}{2}(3+2i)$

(c) $\frac{\pi}{3}(3+2i)$

(d) $\frac{\pi}{2}(2+3i)$

12. The integral of the function $\int_C \frac{\sin z}{2z}\,dz$ where C is the unit circle is

(a) π

(b) 2π

(c) 0

(d) 1

13. The integral of the function $\int_C \frac{e^z}{z}\,dz$ where C is the unit circle is

(a) $2\pi i$

(b) $-\pi i$

(c) πi

(d) $-2\pi i$C

14. The integral of the function $\int_C \frac{dz}{3z^2+1}$ where C is the circle $|z| = 1$ is

(a) π

(b) $-\pi$

(c) 0

(d) 1

15. The integral of the function $\int_C \frac{z+2}{z-2}\,dz$ where C is the unit circle is

(a) 0

(b) 1

(c) -1

(d) π

16. The integral of $\int_C \bar{z}\,dz$ where C is the upper half of the circle $|z| = 1$ from $z = -1$ to $z = 1$

(a) πi

(b) $-\pi i$

(c) $2\pi i$

(d) $-2\pi i$

17. If f is analytic within and on a simple closed positively oriented contour C and if z_0 is a point interior to C, then $\int_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz$ equals

(a) $\frac{n!}{2fi} f''(z_0)$

(b) $\frac{2fi}{n!} f''(z_0)$

(c) $\frac{2fi}{(n+1)!} f''(z_0)$

(d) $\frac{2fi}{n+1} f''(z_0)$

18. If f is continuous in a domain D and if $\int_C f(z)\,dz = 0$ for every simple closed positively oriented contour C in D, then

(a) f is analytic in D

(b) f is real valued in D

(c) f is constant in D

(d) f is imaginary in D

19. The converse of Cauchy- integral theorem is

(a) Euler's theorem

(b) Liouville's theorem

(c) Morera's theorem

(d) Goursat's theorem

20. Piecewise smooth curve is also known as

(a) contour

(b) smooth curve

(c) circle

(d) regular curve

MODULE III

1. Taylor series representation for $1/z$ about $z = 1$ is

(a) $1 + (z-1) + (z-1)^2 + (z-1)^3 + \cdots$

(b) $1 - (z-1) - (z-1)^2 - (z-1)^3 - \cdots$

(c) $1 - (z-1) + (z-1)^2 - (z-1)^3 + \cdots$

(d) $1 + (z+1) + (z+1)^2 + (z+1)^3 + \cdots$

2. A Maclaurin series is a Taylor series with center

(a) $z_0 = 0$

(b) $z_0 = 1$

(c) $z_0 = 2$

(d) $z_0 = -1$

3. Maclaurin series of $sin\, z$ is

(a) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{z^{2n+1}}{(2n+1)!}$

(b) $\sum_{n=0}^{\infty}(-1)^{n}\frac{z^{2n+1}}{(2n+1)!}$

(c) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{z^{2n}}{(2n)!}$

(d) $\sum_{n=0}^{\infty}(-1)^{n}\frac{z^{2n}}{(2n)!}$

4. The power series representation of $\frac{1}{1-z}$ in non-negative powers of z is

(a) $1 + z + z^2 + z^3 + \cdots$

(b) $1 - z + z^2 - z^3 + \cdots$

(c) $1 + z + z^3 + z^5 + \cdots$

(d) $1 - z + z^3 - z^5 + \cdots$

5. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ is

(a) 1

(b) 0

(c) -1

(d) ∞

6. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$ is

(a) 1

(b) $1/e$

(c) e

(d) 0

7. Find the radius of convergence of the series $\sum_{n=0}^{\infty} n^n z^n$ is

(a) 0

(b) 1

(c) e

(d) ∞

8. The center of the power series $\sum_{n=0}^{\infty} (z + 4i)^n$ is

(a) $4i$

(b) $2i$

(c) 4

(d) $-4i$

9. A power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ always converges for

(a) at all z which are either real or purely imaginary

(b) at least one point z

(c) at all z with $|z - z_0| < R$ for some $R > 0$

(d) all complex numbers z

10. The center of the power series $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$ is

(a) 1

(b) -1

(c) 0

(d) 2

11. If the principal part of $f(z)$ at z_0 is zero, then the point z_0 is known as
    (a) pole
    (b) removable singular point
    (c) simple pole
    (d) None of these
12. The zero of the function $\frac{z}{\cos z}$ is
    (a) 1
    (b) 0
    (c) -1
    (d) π
13. The singularity of the function $\frac{e^z-1}{z}$ is
    (a) π
    (b) $-\pi$
    (c) 1
    (d) 0
14. The order of the zeros of the function $\frac{\sin z}{z+4}$ is
    (a) 1
    (b) 2
    (c) 3
    (d) 4
15. The principal part of $f(z)$ at z_0 consists of infinite number of terms, then z_0 is known as
    (a) pole
    (b) essential singular point
    (c) removable singular point
    (d) simple pole
16. The singularity of the function $\frac{e^z}{z^2}$ is
    (a) 0
    (b) 1
    (c) -1
    (d) 2
17. if $f(z)$ has a pole of order m at z_0 then $g(z) = \frac{f'(z)}{f(z)}$ at z_0 has
    (a) a simple pole
    (b) a pole of order m

(c) a pole of order $m+1$

(d) None of these

18. The singular point of the function $\frac{1}{4z-z^2}$ are

(a) $z=0 \ \ and \ \ z=-4$

(b) $z=0 \quad and \quad z=4$

(c) $z=4 \quad and \quad z=-4$

(d) $z=0 \ \ and \ z=2$

19. The power series $b_0+b_1z^{-1}+b_2z^{-2}+\cdots$ converges

(a) Inside of some circle $|z|=R$

(b) on the circle $|z|=1$

(c) on some circle $|z|=R$

(d) outside of some circle $|z|=R$

20. The nature of the singularity of function $\frac{1}{\cos z-\sin z}$ at $z=\frac{\pi}{4}$ is

(a) removable singularity

(b) isolated singularity

(c) simple pole

(d) essential singularity

MODULE IV

1. Which of the following is related to Cauchy residue theorem?

(a) $\int_C f(z)\,dz=0$

(b) $\int_C \frac{f(z)}{z-z_0}\,dz=2\pi i\,f(z_0)$

(c) $\int_C f(z)\,dz=2\pi i(sum\ of\ residues)$

(d) None of these

2. Residue at $z=2$ of the function $\frac{2z+1}{z^2-z-2}$ is

(a) $\frac{5}{3}$

(b) $\frac{1}{3}$

(c) $\frac{3}{5}$

(d) $\frac{2}{3}$

3. Integration of the complex function $\frac{z^2}{z^2-1}$ in the counterclockwise direction, around $|z-1|=1$ is

(a) $-\pi i$

(b) 0

(c) πi

(d) $2\pi i$

4. The residue of the function $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 3$ is

(a) -8

(b) $\frac{101}{16}$

(c) 0

(d) $\frac{27}{16}$

5. Residue of the function $\frac{1}{z+z^2}$ at $z = 0$ is

(a) 0

(b) 1

(c) -1

(d) 2

6. Residue of the function $cot\ z$ at the singular points is

(a) 1

(b) -1

(c) -2

(d) 0

7. Residue of the function $\frac{4}{1-z}$ at the singular points is

(a) 4

(b) -4

(c) 2

(d) -2

8. Residue of the function $\frac{\sin z}{z^4}$ is

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{1}{6}$

(d) $-\frac{1}{6}$

9. What is the residue of the function $\frac{1-e^{2z}}{z^4}$ at its pole?

(a) $\frac{4}{3}$

(b) $-\frac{4}{3}$

(c) $-\frac{2}{3}$

(d) $\frac{2}{3}$

10. The residue of the function $\frac{\tan z}{z^2}$ is

(a) $\frac{4}{\pi}$

(b) $\frac{2}{\pi^2}$

(c) $-\frac{4}{\pi^2}$

(d) $\frac{2}{\pi}$

11. Given $f(z) = \frac{z^2}{z^2+a^2}$. Then

(a) $z = ia$ is a pole and $\frac{ia}{2}$ is a residue at $z = ia$ of $f(z)$

(b) $z = ia$ is a simple pole and ia is a residue at $z = ia$ of $f(z)$

(c) $z = ia$ is a simple pole and $-\frac{ia}{2}$ is a residue at $z = ia$ of $f(z)$

(d) None of the above

12. If C is a circle $|z| = 4$ and $f(z) = \frac{z^2}{(z^2-3z+2)^2}$, then $\oint f(z)\,dz$ is

(a) 1

(b) 0

(c) -1

(d) -2

13. Integration of $\int_C \frac{dz}{z \sin z}$, where C is $x^2 + y^2 = 1$ is

(a) 1

(b) 2

(c) 0

(d) -1

14. The value of $\oint \frac{1}{z^2}\,dz$, where the contour is the unit circle traversed clockwise is

(a) $-2\pi i$

(b) 0

(c) $2\pi i$

(d) $4\pi i$

15. The integral of the function $\int_C e^{1/z}\,dz$ where C is the unit circle counterclockwise direction is

(a) $2\pi i$

(b) 0

(c) πi

(d) $-\pi i$

16. The integral of $\int_C \frac{1}{z \sin z} dz$ where C is the unit circle oriented in the positive direction is

(a) πi

(b) $2\pi i$

(c) $-\pi i$

(d) 0

17. Residue of $\frac{\cos z}{z}$ at $z = 0$ is

(a) 1

(b) -1

(c) 2

(d) 0

18. The integral of $\int_C \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$, in counterclockwise direction is

(a) $2\pi i$

(b) $5\pi i$

(c) $10\pi i$

(d) $-2\pi i$

19. The zero of order is known as

(a) complex zero

(b) simple zero

(c) singularity

(d) None of these

20. Singularities of rational functions are

(a) poles

(b) essential

(c) non isolated

removable

ANSWER KEY

MODULE I

1.c 2.a 3.b 4.b 5.a 6.a 7.a 8.d 9.b 10.d 11.d 12.a 13.a 14.a
15.c 16.b 17.d 18.b 19.a 20.c

MODULE II

1.b 2.a 3.b 4.d 5.b 6.c 7.a 8.d 9.c 10.a 11.b 12.c 13.a
14.c 15.a 16.b 17.b 18.a 19.c 20.a

MODULE III

1.c 2.b 3.b 4.a 5.d 6.c 7.a 8.d 9.b 10.c 11.b 12.b 13.d
14.a 15.b 16.a 17.a 18.b 19.d 20.c

MODULE IV

1.c 2.a 3.c 4.d 5.b 6.a 7.b 8.d 9.b 10.c 11.a 12.b
13.c 14.b 15.a 16.d 17.a 18.c 19.b 20.a

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